Inverse problems for radiative transport equations with the whole velocity domain

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Abstract

Let Ω be a smooth bounded domain of $\mathbf{R}^n$, $n \geq 2$. Let V be a domain in $\mathbf{R}^n$ with $0 \notin \overline{V}$, where $\overline{V}$ is the closure of V. Let $\nabla = {}^t(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n})$ and $v \cdot v'$ denote the dot product of vectors v, v'. Let $u(x,v,t)$ be the specific intensity, which obeys the following radiative transport equation:

$$\partial_t u(x,v,t) + v \cdot \nabla u(x,v,t) = \sigma(x,v)u - \int_V k(x,v,v')u(x,v',t)\, dv',$$

for $x \in \Omega$, $v \in V$ and $0 < t < T$,

$$u(x,v,0) = a(x,v), \quad x \in \Omega,\ v \in V,$$

with suitable boundary condition on subboundary $\Sigma \times (0,T) \subset \partial\Omega \times \partial V \times (0,T)$.

We consider the inverse problem of determining the coefficient σ by extra boundary data of u on $(\partial\Omega \times \partial V) \setminus \Sigma) \times (0,T)$ by single measurement data after suitable chosen initial value a.

The method by Bukhgeim and Klibanov yields the uniqueness and the stability for this inverse problem and we refer to Klibanov and Pamyatnykh [1, 2] and Machida and Yamamoto [3]. However in the existing papers, one cannot consider an arbitrary range of velocities.

Here we establish a new Carleman estimate to prove the the uniqueness and the stability for this inverse coefficient problem for general V which is far from 0.

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References

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