Direct linear inversion for discontinuous elastic parameters recovery from internal displacements

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Abstract

In this talk, I will present a study of the invertibility and the corresponding stability for the elastography problem from internal data. In medical imaging, it is possible to track the inner fast displacement field of a living tissue using MRI, Optical Coherence Tomography or Ultrafast Ultrasound Imaging. From this data a major problem is to provide a stable and fast method to recover elastic properties of the biological tissue. The displacement field can be generated either by static or dynamic (in time regime or time harmonic regime) solicitations or even by natural sources (heart beats, breathing,...). Most of the time these external forces are not accurately known.

In order to avoid iterative inversion procedure, we propose a direct local and linear approach in looking for the inversion the stiffness-to-force operator. If $\mathbf{u}(x)$ is the inner displacement field, the associated stiffness-to-force operator is given by

$$A_{\mathbf{u}} : \mathbf{C} \longmapsto -\nabla \cdot (\mathbf{C} : \nabla^s \mathbf{u}). \tag{1}$$

I will present a general approach to numerically invert this kind of linear operators without neither smoothness hypothesis on the unknown tensor $\mathbf{C}$, nor boundary knowledge. I will then discuss the general stability question linked to the closed range property of the linear operator $A_{\mathbf{u}}$.

In a second time, I will focus on the most useful question that is the shear modulus reconstruction. In this case, I will show that under non restrictive piecewise smoothness hypothesis, the inversion is possible with only one measurement. I will then give corresponding stability results in L^2.